Lab 1 Final Report

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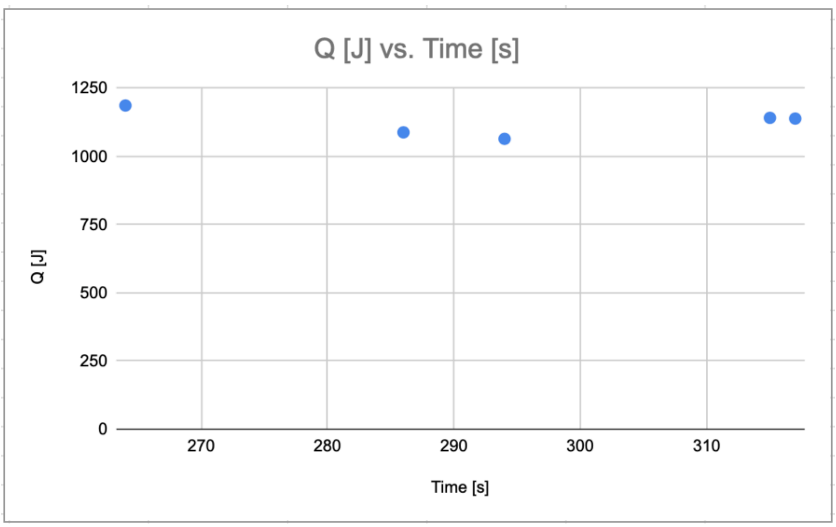
Activity 1: Analytical solution

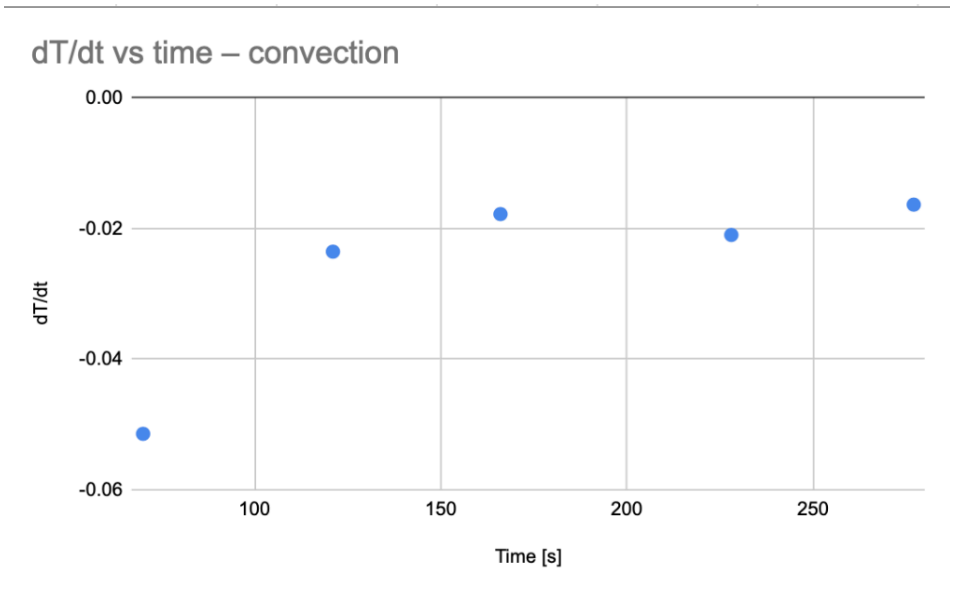
After reviewing the equations for the different forms of heat transfer provided, we determined that we would need to gather data and model for heat gained through conduction between the heating element and the water, heat lost through convection between the water and kettle, and heat lost through radiation between the kettle and the atmosphere.

First we ran a number of different experiments to gather data related to these different modes of heat transfer, and the results from this data collection can be seen below;

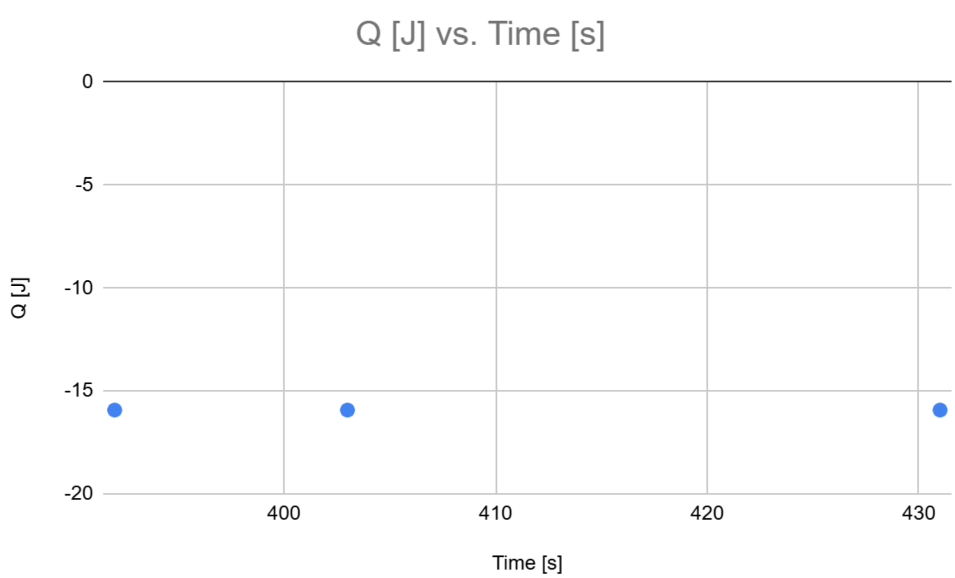
Starting Temp [C]	Target Temp [C]	Final Temp [C]	Time [s]	Calcd Power [W]	Obsvd Power [W]		ΔT / Δt
7.1	87.8	93	315	1072.413333	1141.515556		0.2726984127
7.1	93.33	96.4	264	1367.268106	1415.946212	Did not let kettle	0.3382575758
11.3	87.8	91.4	294	1089.214286	1140.471429		0.2724489796
14.5	93.33	96.4	286	1153.784545	1198.718182		0.2863636364
12.4		98.1	317				0.2703470032



To collect the data for heat gained through conduction illustrated in the graph above we measured initial water temperature, set the kettle to a target temperature, measured elapsed time, then measured final temperature. The heat gained through conduction seen here is much larger than the heat losses due to convection and radiation.

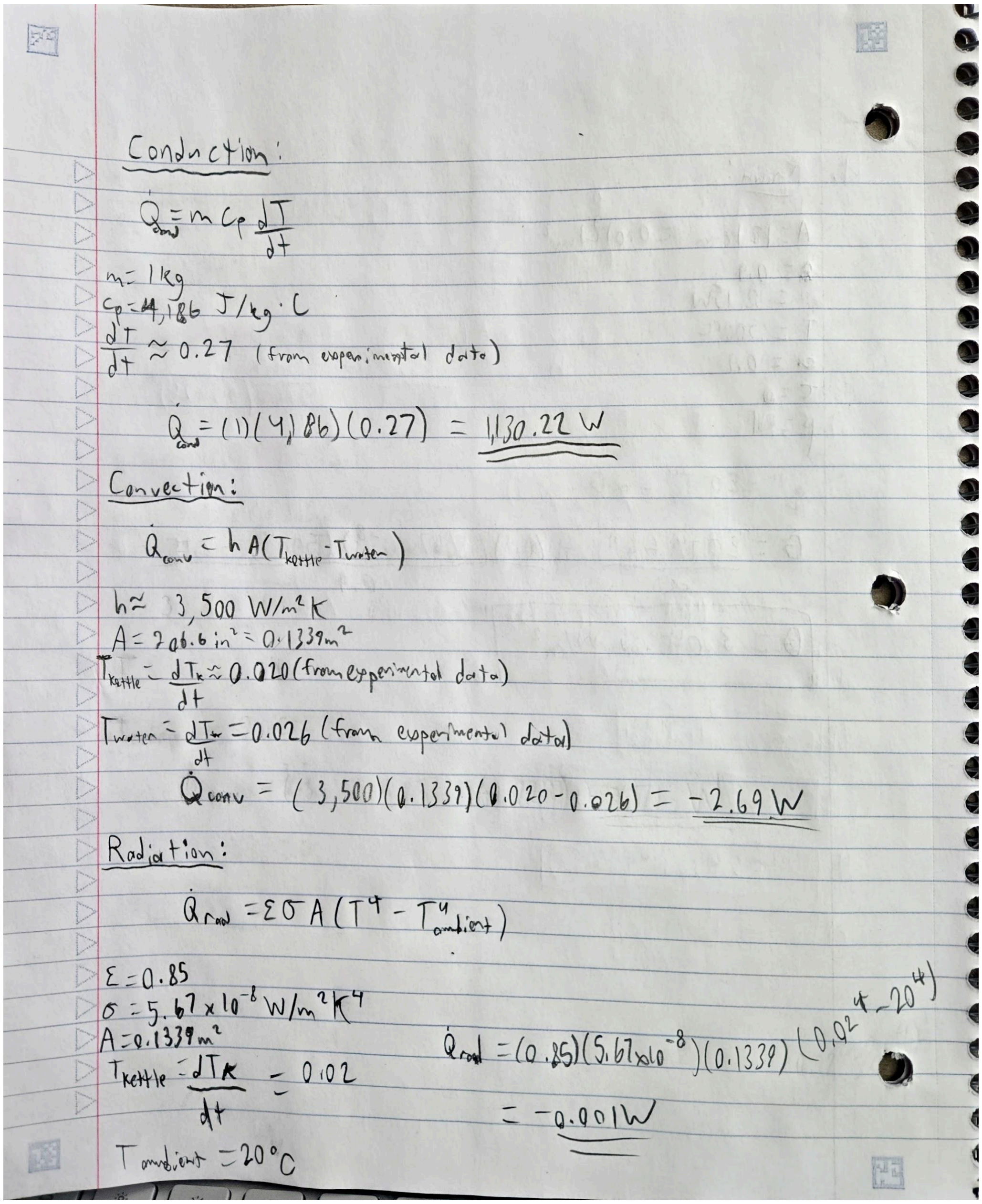


The graph shown above depicts the dT/dt vs time data we collected for our convection data collection experiments, which consisted of boiling water and then observing change in temperature of the water and the kettle over time. Heat lost due to convection in this scenario is dependent on dT/dt, so we can assume this heat loss is fairly negligible because we are not seeing temperatures changes beyond 0.06 [ºC] per second. These heat losses to convection may be something we would have considered more if the process of these experiments and/or heating up water for tea were a longer process.



Lastly, the graph shown above was made from data collected during tests run to determine heat loss due to radiation. These tests consisted of boiling water and observing change in temperature of the kettle over time. As can be seen from the graph, radiative heat loss practically stayed the same throughout our tests and is also negligible compared to the heat gained during conduction.

Lastly we used data from these tests we ran to model some of these heat gains/losses through hand calculations, and the results can be seen below.



Conduction:

$\dot{Q}_{cond} = m\, c_p \frac{dT}{dt}$

$m = 1 kg$

$c_p = 4{,}186\ J/kg \cdot C$

$\frac{dT}{dt} \approx 0.27$ (from experimental data)

$\dot{Q}_{cond} = (1)(4{,}186)(0.27) = \underline{1{,}130.22\ W}$

Convection:

$\dot{Q}_{conv} = hA(T_{kettle} - T_{water})$

$h \approx 3{,}500\ W/m^2 K$

$A = 206.6\ in^2 = 0.1339 m^2$

$T_{kettle} = \frac{dT_k}{dt} \approx 0.020$ (from experimental data)

$T_{water} = \frac{dT_w}{dt} = 0.026$ (from experimental data)

$\dot{Q}_{conv} = (3{,}500)(0.1339)(0.020 - 0.026) = \underline{-2.69\ W}$

Radiation:

$\dot{Q}_{rad} = \varepsilon \sigma A (T^4 - T^4_{ambient})$

$\varepsilon = 0.85$

$\sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$

$A = 0.1339 m^2$

$T_{kettle} = \frac{dT_k}{dt} = 0.02$

$T_{ambient} = 20°C$

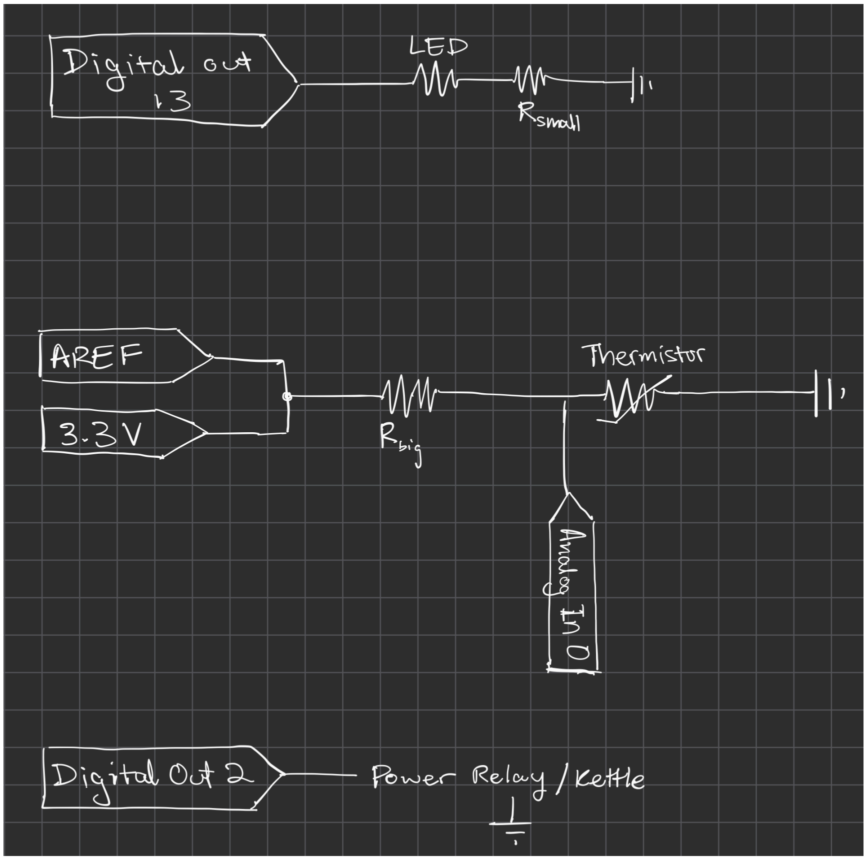
$\dot{Q}_{rad} = (0.85)(5.67 \times 10^{-8})(0.1339)(0.02^4 - 20^4)$

$= \underline{-0.001 W}$

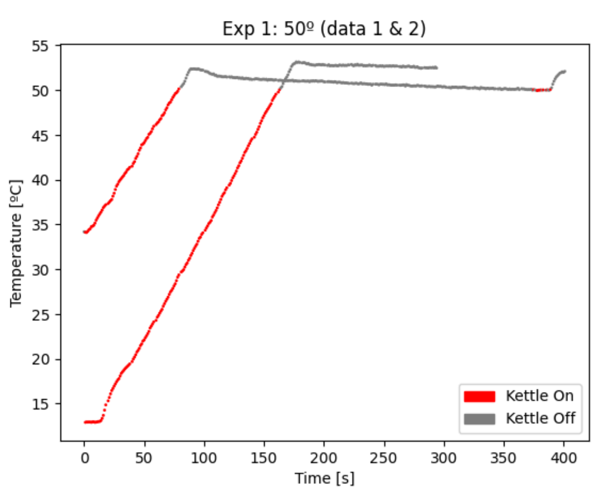
As we assumed from what we saw from the data we collected, the mathematical modeling seemed to further prove that heat losses due to convection and radiation were generally negligible compared to heat gained through conduction. The conductive heat rate was many orders of magnitude higher than convection and radiation in our observed data and calculations.

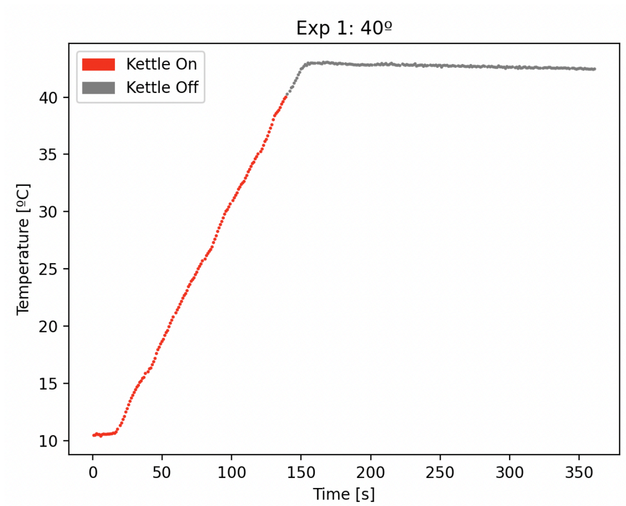
Activity 2: Experimental solution

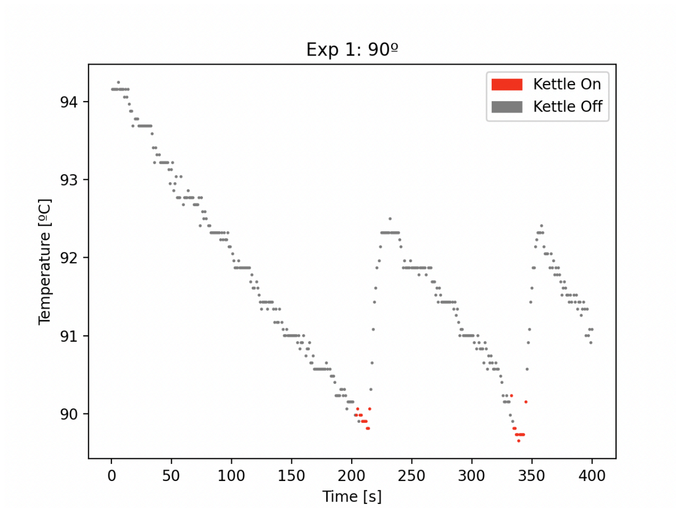
We first assembled the circuit to control an LED light, get readings from a thermistor, and control a relay to turn the kettle on/off.



After calibrating our thermistor and ensuring it read roughly the same temperature as provided thermometers, we wrote a simple control loop. It would turn the kettle and LED (as an indicator) on if the thermistor reading was below a certain threshold, and switch it off once it crossed the threshold. We tested it for different target temperatures, keeping volume constant at 5 cups. The graphs are shown below.



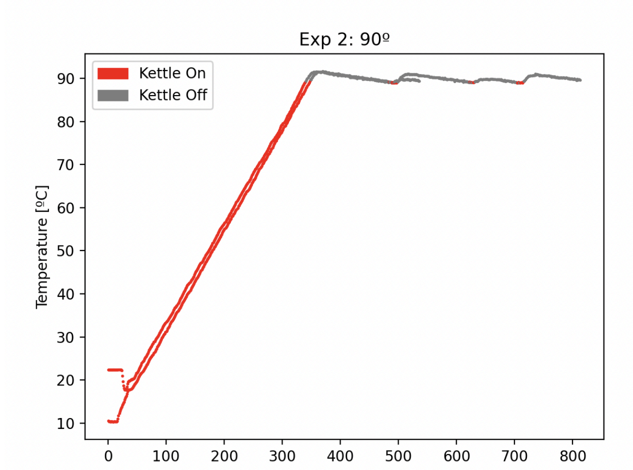




We made a few observations from these graphs that helped us in our second iteration:

- Heating rate is about 3º / 10sec
- Cool-down is
  - 3º / 100sec — hotter target
  - 0.5º / 200sec — cooler target
- Half-period: 20sec → we'd be at target every 20sec
  - If we code threshold to be about 1º below true target, we'd hit target every 20sec, and be ±1º at all times (after initial heat-up)
- Bang-on causes overshoot of about 2.5º
  - In 90deg test, initial overshoot was much higher. If we are given a very short time, we will need to bang-off sooner to minimize this larger overshoot

Because of the consistent 2.5-degree overshoot on the initial heating cycle, we slightly changed the control loop to turn the kettle off when it was 1 degree below the target temperature, instead of at the target. This resulted in the following behavior, which also helped us determine the water would take **about 5-6 minutes to heat up** to a reasonably hot temperature (90-100 degrees):



Slightly undershooting to the target helped the temperature stay within about 1 degree of the target. Our final code can be found in the Appendix.

Activity 3: Simulated solution

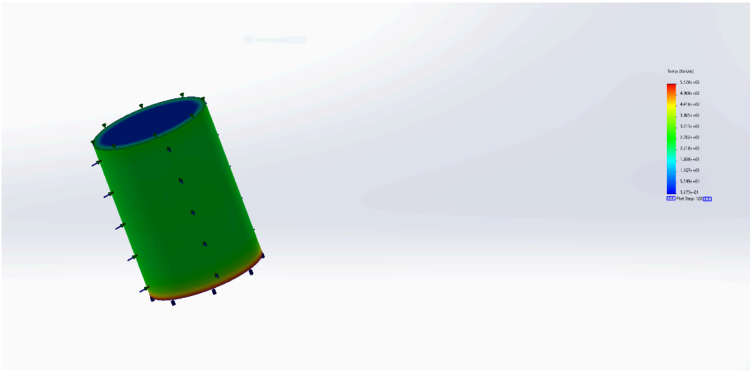
We used Solidworks to simulate the conductive heat transfer during the heating of water in a kettle. Initially, we created a simplified cylindrical part with dimensions approximately like those of the actual kettle (~3” radius, ~8” height). We assigned the cylinder a material of Polypropylene to replicate the kettle's exterior. However, when running the conduction simulation with only this solid cylinder, the results were not meaningful, since we did not include water, and the simulation was ineffective in achieving our objective.

To improve the model, we created a second cylinder with the same dimensions but defined the material as water. A Polypropylene plate was added beneath the water cylinder to serve as the heat source.

In setting up the thermal study, we focused on conduction and convection and neglected radiation. Conduction was applied from the bottom plate to the water, while convective heat losses were applied to the outer walls and top surface to represent the true convective heat loss to the environment. The thermal load parameters included a heat power input of approximately 1300 W, an ambient temperature of 295 K, and a convection coefficient of **h = 40 W/m²K**. We defined the study as transient over a **two-minute** interval, incorporating fluid convection effects. However, during setup, the software prompted us to upload a convection model file. As we were unsure how to access the appropriate file, we proceeded without it.

Despite this, the simulation produced interesting results. By reviewing each time step, we observed the heating process over time. As expected, the highest temperatures were concentrated at the bottom between the heating plate and the water. The results were consistent with heat transfer principles, showing the water temperature increasing from **295 K (room temperature) to approximately 375 K (near boiling)**. The following is the simulation at the first time step:

And the following is a screenshot from the final time step:

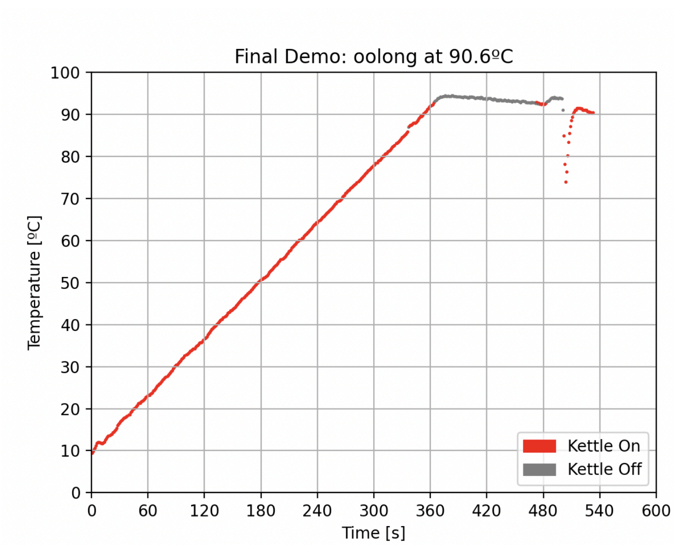


Demo Prediction

Based on the tests we ran in Activity 2, we theorized that we would need to start heating water at least 6 minutes before it was Tea Time. The control loop would keep it within 1 degree of the target temperature, so it did not matter too much if we started heating the water too early.

Final Demo Analysis

For the final demo, we also needed to account for the fact that the water may cool slightly when being poured from the kettle to the cup. Based on empirical tests, we estimated that we should heat the water about 3 degrees higher than what we were currently doing. This ensured that our true final temperature in the cup was at the target. For our final demo, we were within 1 degree of the target (target was 90.6ºC, our highest temperature in the cup was 91.4ºC). The graph of our final demo data is shown below. We started heating our water approximately 7 minutes before our check in time to ensure it would be at the desired temperature at the time our group was assigned.



Appendix: full final code

```
// SPDX-FileCopyrightText: 2011 Limor Fried/ladyada for Adafruit Industries
//
// SPDX-License-Identifier: MIT

// thermistor-2.ino Intermediate test program for a thermistor. Adafruit Learning
System Tutorial
// https://learn.adafruit.com/thermistor/using-a-thermistor by Limor Fried, Adafruit
Industries
// MIT License - please keep attribution and please consider buying parts from
Adafruit

// which analog pin to connect
#define THERMISTORPIN A0
// how many samples to take and average, more takes longer
// but is more 'smooth'
#define NUMSAMPLES 5
// the value of the 'other' resistor
#define SERIESRESISTOR 10800

bool kettleOn;
#define THERMISTORNOMINAL 10000
// temp. for nominal resistance (almost always 25 C)
#define TEMPERATURENOMINAL 25
// how many samples to take and average, more takes longer
// but is more 'smooth'
#define NUMSAMPLES 5
// The beta coefficient of the thermistor (usually 3000-4000)
#define BCOEFFICIENT 3950
// the value of the 'other' resistor
#define SERIESRESISTOR 10000
int samples[NUMSAMPLES];
int ledPin = 13;
int relayPin = 2;
void setup(void) {
 Serial.begin(9600);
 // connect AREF to 3.3V and use that as VCC, less noisy!
 analogReference(EXTERNAL);
 pinMode(ledPin, OUTPUT);
 pinMode(relayPin, OUTPUT);
}
void loop(void) {
```

```
 uint8_t i;
 float average;
 // take N samples in a row, with a slight delay
 for (i=0; i< NUMSAMPLES; i++) {
  samples[i] = analogRead(THERMISTORPIN);
  delay(10);
 }
 // average all the samples out
 average = 0;
 for (i=0; i< NUMSAMPLES; i++) {
    average += samples[i];
 }
 average /= NUMSAMPLES;
//   Serial.print("Average analog reading ");
//   Serial.println(average);
 // convert the value to resistance
 average = 1023 / average - 1;
 average = SERIESRESISTOR / average;
//   Serial.print("Thermistor resistance ");
//   Serial.println(average);


 float steinhart;
 steinhart = average / THERMISTORNOMINAL;     // (R/Ro)
 steinhart = log(steinhart);                  // ln(R/Ro)
 steinhart /= BCOEFFICIENT;                   // 1/B * ln(R/Ro)
 steinhart += 1.0 / (TEMPERATURENOMINAL + 273.15); // + (1/To)
 steinhart = 1.0 / steinhart;                 // Invert
 steinhart -= 273.15;                         // convert absolute temp to C
 //   Serial.print("Temperature ");
 float runtime = millis() / 1000;
 Serial.print(runtime, 1);
 Serial.print(", ");
 Serial.print(steinhart);
 Serial.print(", ");
 Serial.println(kettleOn);
//   Serial.println(" *C");


 float targetTemp = 90.6 + 3;  // 3deg offset for cooldown as we pour from kettle to
cup
 if (steinhart > targetTemp-1){
```

```
    digitalWrite(ledPin, LOW);
    digitalWrite(relayPin, LOW);
    kettleOn = false;
  }
  else{
    digitalWrite(ledPin, HIGH);
    digitalWrite(relayPin, HIGH);
    kettleOn = true;
   }
  delay(1000);
}
```